PRICE AND QUANTITY EFFECTS OF TAX REFORM: AN APPLICATION TO WEST VIRGINIA**

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ABSTRACT

This paper extends a methodology for ascertaining the industry-by-industry effects of tax changes on prices suggested originally by Aaron. The extension utilizes assumptions about the nature of final demand and allows one with an input-output table to derive the total effects of such tax changes in terms of quantities of output as well as employment.

The new methodology is applied to West Virginia which substantially reformed its system of gross receipts taxation. The resulting analysis predicts a modest increase in output (+0.2 percent), and a modest decline in the price level (from −0.1 to −2 percent) as a result of the reforms enacted in 1985, and scheduled to take effect in mid 1987.

1. Introduction

ECONOMIC analysis of tax reform proposals at the national level have typically involved examining: 1) the budgetary or revenue implications compared to current law; 2) possible behavioral adjustments in the marketplace, with attendant implications for real economic growth; and 3) examination of the vertical and horizontal equity implications of such changes. For those proposals which impact primarily on business, analysis usually involves showing the implications of tax changes on tax burdens by industry. The modelling of behavioral reactions to tax policy changes has involved either the use of large-scale econometric models[1] of the national economy, or more recently, the use of computable general-equilibrium models of the economy.[2]

At the regional or state level, inferences about the impact of major tax policy changes have by and large been limited to the results of non-behavioral simulation models based on samples of individual income tax returns available from the Statistics Division of the Internal Revenue Service. Interestingly, the availability of national and state-specific input-output tables, which are as prevalent now in many states as econometric models, has not led to their widespread application to tax policy analysis.[3]

Bahl and Shellhammer (1969) develop a methodology using input-output analysis to examine progressivity and tax exportation issues. Based on observed sectoral tax liabilities, assumed to be borne by purchasers, and an input-output table, the methodology determines the total direct and indirect state tax per dollar of output in each sector. Sectoral export data, together with the forward shifting assumption, allows one to determine how much tax is "exported" (i.e., paid by out-of-state purchasers). Given expenditure by income class data (and maintaining the full shifting assumption), the methodology allows one to gauge the amount of tax borne by the average family in each of the income groups and thus reach conclusions regarding vertical equity.

Melvin (1979) uses a technique similar to one introduced in Aaron (1968), involving an input-output table, to obtain price effects of a value-added tax. Dresch (1977) examines the impact of moving from a national corporate income tax to a value-added tax through the use of a national input-output table.

Virtually all national analyses of tax policy changes have focused on quantity effects (GNP) as contrasted with price effects.[4] This is especially surprising in light of Aaron (1968) which suggested a methodology for inferring price effects of tax policy changes through the use of an input-output table.

The purpose of this paper is to develop a methodology, taking advantage of predicted price changes developed on the basis of Aaron's methodology, which can

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produce forecasts of quantity changes by economic sector. These changes may be taken to be behavioral responses to proposed tax policy changes. In order to do this, we must utilize an input-output table which describes the underlying economic relationships among sectors, and entertain assumptions about the price elasticity of final demand by sector. These assumptions, in conjunction with an input-output table and shifting assumptions required by Aaron's methodology, allow the complete portrayal of price and quantity effects of an economy in response to tax policy changes. We apply this new methodology to alternative proposed reforms of West Virginia's business taxes.

The outline of the paper is as follows: Section 2 briefly reviews the nature of an input-output table, and provides a compact statement of the assumptions and predictive capability of input-output analysis. Also, Aaron's methodology and Dresch's utilization of this methodology are reviewed in order to motivate the methodological contribution of this paper. Section 3 develops these quantity effects per se. Section 4 describes the major components of West Virginia business tax law, the method of updating the 1975 West Virginia input-output matrix, and aggregate and sectoral effects of tax policy changes on prices, quantitites, and manpower requirements. Section 5 concludes.

2. Aaron's Analysis of the Price Effects of Tax Changes

We provide in this section Aaron's derivation of price effects which result from tax policy changes. Aaron's model involves three time periods during which firms adjust their output prices in response to tax changes. More specifically, in period one, the firm is in equilibrium in terms of its factor and product market decisions. In period two, tax law changes, e.g. a tax decrease occurs, and the firm experiences an instantaneous increase in profits. At the end of period two, the firm is at the same output level and utilizes the same levels of factors. All prices are unchanged. The only difference in the firm's circumstances is that it is now in a more profitable position as a consequence of the hypothesized tax decrease.

In period three, the firm responds to its more profitable position, and alters its output prices in accordance with the following parametric behavioral postulate:

> The firm alters its output prices in period 3 so that before-tax profits are αT lower than in period 2, where T is the increase in its after-tax profit from period 1 to period 2.

Here, α is the so-called "shifting parameter" introduced by Aaron to characterize the willingness of a firm to pass on a portion of tax savings or increases to its customers.[5]

By assuming that an economic sector behaves as the above described firm behaves, Aaron is able to readily utilize inter-industry flows described by an input-output table and therefore make statements about the economy-wide price effects of a tax change.

This paper makes use of the following notation:

- $\langle v \rangle$: $\langle v \rangle$ denotes the matrix with the vector v along the diagonal and zeroes elsewhere.
- ι: ι is the unit vector containing only 1's.
- p: The superscript p denotes either b for "before the tax change," or a for "after the price effects." The prefix N generally denotes normalization, as shown in the definitions.
- X: X_i^p is gross sales by sector i.
- M: M_{ij}^p is dollars of sector i's output purchased by sector j.
- NM: $NM_{ij}^p = M_{ij}^p / X_j^p$
- T: T_j is tax reduction in sector j.
- NT: $NT_j = T_j / X_j^b$
- s: s is a vector of price effects by sector, and is derived by application of Aaron's technique. Explicitly, s_i times the old price of any product of sector i is its new price.
- V: V_j^p is the sum of imports by sector j and the value added of sector j, where value added is defined as the sum of wages, return to capital, rent, interest, and taxes.

FD: FD_i is final demand in sector *i* before the tax change.
NV: $NV_j^p = V_j^p / X_j^p$
α: α_i is the shifting parameter for sector *i*.
I: The identity matrix with the same dimensionality as M.

A careful review of Aaron's contribution suggests the following behavioral assumptions are necessary to his analysis and will be entertained below:

1. The behavioral postulate stated above: A firm in sector *i* alters its output prices in period 3 so that before-tax profits are $\alpha_i T$ lower than in period 2, where T is the increase in its after-tax profit from period 1 to period 2.
2. As implied by the definition of *s*, the price increases of all the goods produced in any given sector are uniform.
3. All *quantities* of goods, services and labor exchanged are unaffected by the price change.
4. Only the prices of sector outputs change. Thus, wages and import prices are unchanged.

With these assumptions, price effects can be uniquely determined. Of related interest at this point in our review of Aaron is the work of Dresch et al., which modifies Aaron's methodology by stipulating a somewhat different assumption about the firm's response to tax changes. Recall that in period 3, firms adjust their output prices in order to meet a particular before-tax profit target. Dresch suggests in contrast that firms set prices to achieve a particular level of *after-tax* profit. Dresch's assumption requires that one simulate the new tax system in order to analytically derive tax liabilities within the regime, resulting from price changes. As a practical matter, this may not always be empirically feasible.

Table 2-1 displays by time period the impact of tax change on before- and after-tax profits under Aaron's behavioral postulate. Note that the status of after-tax profit in the third period is unknown, in contrast with Dresch, who makes this level the firm's objective in changing prices.

TABLE 2-1
ILLUSTRATION OF BEHAVIORAL POSTULATE

Period:	1	2	3
Before-tax profit:	X	X	$X - \alpha T$
After-tax profit:	Y	Y + T	?

Before-tax profit in period 3 is the essence of Aaron's assumption.

3. Quantity Effects of Price Changes Due to a Tax Change

It may be expected that changes in product market prices will lead to changes in final demand for such commodities. With this in mind, we add a fourth time period to the three described in section 2 above and introduce the following additional postulate:

5. New prices resulting from adjustments in period 3 remain in place, and final consumers (households and government) respond to the price changes by altering their physical consumption levels. Sector outputs adjust as necessary to meet final demands, taking inter-sector flows into consideration.

In effect, by knowing the elasticity of final demand, and the change in prices being offered to the marketplace by producers, we may move down the demand curve for final products and identify the new quantities of output being demanded. By entertaining relations between factor utilization and these new output levels, we may then be able to make statements about subsequent capital and labor utilization levels in the economy. Of policy interest is the comparison of revenues lost initially to these "real" effects derived through the input-output table.[6]

By postulating a vector η of demand elasticities, we can derive the changes in final demand resulting from the price changes. Given the new levels of final demand, we may then go on to determine their impact on gross outputs, X, in the traditional fashion of input-output analysis, using the total requirements table. The final result in terms of our basic data variables is:[7]

$$X = (I - NM^b)^{-1} \langle FD \rangle \langle\langle \eta \rangle\rangle \langle\langle NV^b - \langle \alpha \rangle NT) (I - NM^b)^{-1} \rangle\rangle \iota$$

Note that the above is in old (period one) prices.

Recall the assumptions implicit in this equation:

- Real (physical) input coefficients are unchanged for all domestic producers; and,
- the change in demand is entirely met by domestic producers.

4. An Application to West Virginia: Data, Proposed Tax Changes, and Empirical Results

4.1. Data

Following a recent study by one of the authors,[8] substantial information on the inter-industry pattern of taxes under West Virginia tax law, as well as under several alternatives considered by the West Virginia legislature, are available for analysis. West Virginia is also of interest to this study because of the availability of input-output tables describing the state's economy. West Virginia's business tax structure has been remarkable because of its reliance on a series of taxes imposed on gross turnover. The Business and Occupation Tax (B&O Tax) has been an important part of West Virginia's fiscal landscape since the 1930s.[9]

As a result of the legislative process leading to the substantial reform of West Virginia's business taxes by industry, tables are available describing projected gross outputs and state business taxes at FY 1986 levels, as well as the distribution of state business taxes under two major policy options which are described below.[10]

The first legislative alternative actively considered was the continuation of the phased 5 percent/year rate reduction in the Business and Occupation Tax and the elimination of certain surcharges on banks and the corporate net income tax, contemplated in 1983 under Senate Bill 310, and scheduled to take effect in FY 1986. The second alternative actively considered, and put into law, was House Bill 1693, as amended by the Senate and the Committee on Conference. Under this alternative, the gross receipts taxes on all industries except natural resources and public utilities are eliminated and replaced with a business franchise tax based on apportioned net equity. Also, the corporate net income tax base and rate are respectively broadened and increased. This alternative eliminates the cascading effects of the B&O Tax which has been criticized for many years.[11]

By utilizing the simulated patterns of tax liabilities for FY 1985 law, and the two alternatives above described, information on gross receipts by industry, and an input-output table (described below) of the West Virginia economy, we are able to apply the above-described methodology to ascertain the price and quantity effects of the various policy alternatives.

The first West Virginia input-output table was developed by Miernyk (1969), and a subsequent one for 1975 was developed by Loviscek et al. (1979). In this analysis we employ the 1975 input-output table, updated to describe the FY 1986 economy.

It may seem inappropriate to "update" an input-output matrix and thereby alter input coefficients, since this is at variance with an assumption which input-output analysis often loosely connotes: that of fixed physical ratios among the inputs and outputs of a firm or sector. However, a close examination of Leontief's original work indicates that at least Leontief believed that it is appropriate to expect these ratios to change over time. In fact, the postulated relationship between input-output matrices for different periods which is at the core of the *RAS* technique was first proposed by Leontief.[12]

Technically, the *RAS* method requires row-sum and column-sum constraints on the matrix to be estimated; that is, total intermediate inputs by sector, and total intermediate outputs. In this study we are able to estimate these by appealing to the stability of these figures, by sector, as proportions of gross sales. Our intermediate figures are based on the assumptions that intermediate input proportions

Table 4-1: Aggregate Effects

	Senate Bill 310		Conference Agreement	
	η'	$\eta=1$	η'	$\eta=1$
Panel A: a'				
Price:	99.861	99.861	99.775	99.775
Quantity:	100.217	100.139	100.270	100.225
Manpower:	100.199	100.197	100.121	100.135
Panel B: a^2				
Price:	99.870	99.870	99.817	99.817
Quantity:	100.205	100.130	100.227	100.183
Manpower:	100.185	100.185	100.064	100.072

in FY 1986 are identical to those in 1975, and intermediate output proportions are identical save for a scalar adjustment.[13] This enables us to apply the *RAS* method to estimate the full FY 1986 matrix.

In summary, the steps necessary to implement the empirical application of Aaron's methodology and that being proposed by the authors are as follows:

1. Based on projected FY 1986 gross sales by sector, update the 1975 West Virginia input-output table published in Loviscek et al. (1979) to obtain an FY 1986 input-output table, using the RAS method;
2. Postulate combinations of Aaron's α parameter by sector and obtain estimated price changes per sector;[14]
3. Postulate, based on surveys of empirical demand studies[15] and inspection, elasticities of final demand by sector, to obtain new final demands by sector;
4. Apply the traditional input-output calculation to derive gross output effects by sector;[16]
5. Derive employment effects, assuming fixed manpower: sectoral output relationships.

4.2. Empirical Results: Economy-Wide and Sectoral Effects

We report our empirical results in two forms.[17] First, we provide economy-wide results which indicate price, quantity, and manpower effects which result from various combinations of shifting and elasticity assumptions in conjunction with two major policy alternatives to current law.

Second, we display, for important economic sectors of the West Virginia economy, the price, quantity, and manpower effects of various alternatives to current West Virginia tax law. We focus in particular on Wholesaling, Retailing, Coal, Chemicals, and Electrical Utilities, which

Table 4-2: Sectoral Effects

	Senate Bill 310			Conference Agreement		
	% Tax Liability Change	Price Index	Quantity Index	% Tax Liability Change	Price Index	Quantity Index
Panel A: a^1, η^1						
2. Coal,Mining	-6.616	99.813	100.388	-1.874	99.935	100.098
26. Wholesale	-10.243	99.950	100.094	+28.531	100.093	100.024
43. Electr Util	-7.940	99.761	100.537	+5.175	100.112	99.785
29. Retail	-9.202	99.903	100.069	-24.855	99.814	100.116
14. Chemicals	-8.160	99.925	100.152	-13.570	99.896	100.213
Panel B: a^1, η=1						
2. Coal,Mining	-6.616	99.813	100.189	-1.874	99.935	100.054
26. Wholesale	-10.243	99.950	100.082	+28.531	100.093	99.999
43. Electr Util	-7.940	99.761	100.222	+5.175	100.112	99.928
29. Retail	-9.202	99.903	100.101	-24.855	99.814	100.185
14. Chemicals	-8.160	99.925	100.077	-13.570	99.896	100.109
Panel C: a^2, η^1						
2. Coal,Mining	-6.616	99.815	100.384	-1.874	99.943	100.082
26. Wholesale	-10.243	99.951	100.090	+28.531	100.094	100.009
43. Electr Util	-7.940	99.763	100.532	+5.175	100.114	99.778
29. Retail	-9.202	99.907	100.067	-24.855	99.826	100.108
14. Chemicals	-8.160	99.927	100.147	-13.570	99.906	100.191
Panel D: a^2, η=1						
2. Coal,Mining	-6.616	99.815	100.187	-1.874	99.943	100.046
26. Wholesale	-10.243	99.951	100.078	+28.531	100.094	99.984
43. Electr Util	-7.940	99.763	100.220	+5.175	100.114	99.923
29. Retail	-9.202	99.907	100.097	-24.855	99.826	100.171
14. Chemicals	-8.160	99.927	100.074	-13.570	99.906	100.097

Note: Tax liability change is shown as the change in liability as a percentage of tax liability in FY 1986 when compared to FY 1985 law.

are important economic sectors in the West Virginia economy. In calculating price effects, it will be convenient to examine the overall effects of tax changes on the Laspeyres' price index for final demand.

We entertain two sets of shifting assumptions in order to implement the above methodology. It should be noted that these assumptions do not exhaust the range of shifting assumptions which one might wish to investigate. However, since our objective in this paper is primarily methodological, we will limit our empirical analysis to the following shifting assumptions: complete forward shifting of tax changes,[18] and industry-specific shifting assumptions due to Leyden (1976). In order to test the sensitivity of our elasticity of final demand assumptions, we calculate results assuming a uniform elasticity of -1.0 across all sectors as well as employing our best estimates of the elasticity of final demand by sector.

Table 4-1 displays the price, quantity, and manpower effects of Bill 310 and of the final legislation under our alternative shifting and elasticity assumptions. Since the elasticity effects impact only on the quantity and manpower variables, the price effects will not vary across elasticities. Of immediate interest is the fact that under either shifting assumption, the aggregate price level in the West Virginia economy declines by between .13 to .22 percent. under the 100 percent shifting assumption (see panel A) and the preferred elasticity of final demand assumption (see columns headed η^1), Senate Bill 310's tax changes would be accompanied by a .22 percent increase in quantity, while under the actual legislation, there would be a .27 percent increase in quantity. Under Leyden's shifting assumptions (see panel B), the quantity effects are in the same direction: for Senate Bill 310, quantity increases .21 percent, while under the actual legislation quantity increases .23 percent.

The manpower effects of the two policies under the various behavioral assumptions, however, differ somewhat from the quantity effects. Employment rises under both scenarios; however, it rises somewhat more under Senate Bill 310.

Overall, the effects of the policy changes are rather modest, since changes in prices, quantities, and manpower are always in the aggregate less than 1 percent. Of course, to the extent such tax reductions affect the incentive structure in the West Virginia economy beyond that captured by the input-output table, there may well be further, larger, and differential effects of both tax changes, which might more favorably impact on quantities produced and labor requirements.

Table 4-2 displays the impact on important sectors of the West Virginia economy of the two tax changes under various shifting and demand assumptions, as before. The first column in each of the panels of Table 4-2 shows the percentage change in tax liability compared to FY 1985 law operating in the FY 1986 economy. Under Senate Bill 310, each of the five industries experiences a tax reduction of from 6.6 percent to 10.2 percent. As a result, the price level in each industry declines, and the quantity index increases. Note, however, that the price declines and quantity increases are very modest—price changes range from $-.24$ percent to $-.08$ percent, while quantity increases range from .09 percent to .54 percent. On the other hand, the legislation enacted imposes tax increases on wholesaling of 28.5 percent, and on electric utilities of 5.2 percent. Accordingly, these industries experience increases in their price levels of .09 percent to .11 percent, and quantity declines of .08 percent.

5. Conclusions

We have sought in this paper to extend Aaron's methodology to include quantity effects on a known economy which is described by an input-output table. By assuming that the elasticity of final demand is known by economic sector, we are able to describe not only the price effects of various proposed tax policy changes on a regional economy, but also the net quantity effects which such price changes may be expected to induce.

Application of this methodology to the West Virginia economy provides some interesting insights. Substantial tax changes

are found to induce relatively modest, but nonetheless important, price and quantity effects across the West Virginia economy. Further research along the lines being proposed here should include the specific estimation of final demand elasticities for a regional economy, as well as the statistical estimation of specific shifting parameters. Also, the influence of an altered government revenue structure on final demand by government should be investigated, since it is salient here insofar as it impacts on aggregate final demand. It may also be desirable to attempt to distinguish the elasticity of demand by external and domestic consumers. What the analysis contained in this paper has demonstrated is that meaningful quantity implications may be derived by an input-output analysis of proposed tax policy changes. In view of the active national discussions of major tax reform, such analysis may also be of interest at the national level.

FOOTNOTES

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[1]For a discussion of the range of behavioral response which may be inferred from large-scale econometric models, see Congressional Budget Office (1980). It provides a review of the various impact multipliers contained in the major, commercially available macroeconomic models.

[2]See for example Shoven et al. (1984) for a review of the use of computable general equilibrium models for tax policy analysis.

[3]It should be recognized, however, that national, regional and state-specific input-output tables have been used extensively to analyze the impact on gross outputs that result from exogenous changes in final demand, e.g., changes in government spending policy. Examples of this type of analysis may be found in Ballard, Gustely, and Wendling (1980).

[4]Leyden (1976) examines price effects within the context of the West Virginia economy, and in conjunction with an input-output table. However, it is unclear whether or not Leyden utilizes Aaron's earlier methodology. Also, Leyden does not deal with price-induced output effects which are a major contribution of this paper.

[5]The shifting behavior of firms reacting to a change in tax liabilities was estimated in Krzyzaniak (1963).

[6]It would be also desirable to be able to make statements about how subsequent revenues respond to the new tax regime. This is in effect what Dresch achieves, although he does not identify a real quantity effect in the process. Because we are considering a set of taxes in our empirical example below which do not in their entirety depend only on gross output, we are not able to make tax revenues endogenous, and therefore appeal to a postulate specifying gross profit firm targets.

[7]See Appendix II for the algebraic derivation of the given expression.

[8]See Strauss (1984) for a description of alternative business tax reform proposals to West Virginia's system of gross receipts taxation.

[9]For a more complete description of West Virginia tax law and the empirical characteristics of industrial tax liability, see Strauss (1984), Chapters 4 and 5.

[10]The state business taxes under consideration are: the Business and Occupation Tax, the Corporation Net Income Tax, and the Carrier Income Tax.

[11]Elimination of the cascading effects of the gross receipts taxes has been recommended by a number of students of West Virginia tax law. See for example Alvis (1970), Hanczaryk and Thompson (1958), Papke (1966), or Strauss (1984).

[12]Bacharach (1970, p 10) clarifies the constancy assumption: "Leontief himself [Leontief (1951)] thought of such changes as virtual changes; the statement was: had the j'th output been different, the (i,j)th input-output ratio would have been the same. They were not to be thought of as changes through time. Much of Part II of [Leontief (1951)] is in fact devoted to justifying a specific hypothesis on temporal changes in input-output coefficients [. . .] It seemed, however, a small and inoffensive step from an assumption of coefficients' invariance with respect to virtual changes in outputs to one of their invariance with respect to changes in outputs occurring within short periods of time. Such a hypothesis—of temporal as well as virtual invariance—would at once make input-output a powerful tool for the formation of policy [. . .]"

[13]The scalar adjustment to the vector of row-sum constraints is necessary to ensure that their sum is equal to the sum of column-sum constraints. This is required for the internal consistency of the assumptions. Note that while the RAS method has achieved widespread acceptance, it should not be relied upon for certain categories of matrix-updating problems. Hewings (1977) reports that although RAS is successful in updating a matrix for a given region to describe the same region over a later period, it performs poorly when it is used to alter a matrix for one region, to describe a different region. Using survey-based matrices for Washington (1963) and Kansas (1965), Hewings measures, by means of a deviation-frequency technique (it is unclear whether the frequency count for a sector pertains to the corresponding row or column of the matrix), the accuracy of the RAS-estimated Kansas matrix; the conclusion is that it is no closer to the true Kansas matrix than is the Washington matrix it is based on.

Malizia (1974) questions the RAS method even as a means of temporal matrix adjustments. In an evaluation using survey-based matrices for Washington for 1963 and 1967, Malizia et. al conclude the method fails to produce accurate coefficients, but "appears

sufficient for making short-term forecasts of main aggregates."

Other techniques besides RAS have been proposed for situations in which additional information about the current period is available. For example, Morrison (1980) discusses a method of imposing further constraints to exploit additional information which may be available to the analyst, such as constraints on the sum of a subset of matrix entries.

[14]These combinations are based on Leyden (1976).

[15]See, for example, Deaton (1974).

[16]Note that this step and the following one are performed for each tax proposal and combination of shifting and elasticity parameters.

[17]All data used in this project, including the 45-sector input-output table estimated for FY 1986, and APL software used to generate the results, is available from the authors.

[18]The assumption of complete forward shifting is one of the shifting assumptions entertained by Aaron in his work, as well as other researchers. We choose this assumption as our point of departure.

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APPENDIX I

I. Sector Codes, Shifting, Elasticity, Law, and Employment Vectors

Table I-1: Data and Parametric Assumptions

Industry	a^1	a^2	η'	Tax Reduction As % Of Sales		
				310	Base Conference Agmt.	Employment
1. Agric, forestry&fish	1.00	0.75	1.00	0.06	-0.03	1394
2. Coal, other mining	1.00	1.00	2.00	0.16	0.05	61406
3. Petroleum&natural gas	1.00	1.00	2.00	0.25	0.48	6473
4. Gen contractors:bldg	1.00	0.50	0.75	0.15	1.66	7500
5. Gen contrct: nonbldg	1.00	0.50	0.75	0.16	1.55	7194
6. Special trade contrct	1.00	0.50	0.75	0.16	1.01	9676
7. Food&kindred: other	1.00	0.50	0.75	0.06	0.06	1762
8. Food&kindred: dairy	1.00	0.50	0.75	0.04	-0.11	1762
9. Food&kindred: beef	1.00	0.50	0.75	0.07	0.04	1762
10. Apparel&access(manuf)	1.00	0.50	0.75	0.07	0.30	4468
11. Logging & sawmills	1.00	0.75	2.00	0.13	0.49	5389
12. Furniture & wood	1.00	0.75	2.00	0.08	0.52	972
13. Printing & publishing	1.00	0.75	2.00	0.16	-0.13	4550
14. Chemicals (manuf)	1.00	1.00	2.00	0.04	0.07	17656
15. Petroleum (manuf)	1.00	0.75	2.00	0.06	0.38	786
16. Glass (manuf)	1.00	0.50	2.00	0.05	0.17	7649
17. Stone&clay (manuf)	1.00	1.00	2.00	0.05	0.38	7649
18. Primary metal prods	1.00	1.00	2.00	0.03	0.38	22155
19. Fabricated metal	1.00	0.50	2.00	0.10	0.21	8758
20. Machinery exc electr	1.00	0.50	2.00	0.05	0.03	5793
21. Electr. machinery	1.00	0.50	2.00	0.05	-0.10	4263
22. Transportation equip	1.00	1.00	2.00	0.03	0.24	2380
23. Instruments&rel prod	1.00	0.75	2.00	0.03	0.06	1304
24. All other manuf.	1.00	0.75	2.00	0.09	-0.20	12804
25. Eating&drinking ests.	1.00	1.00	0.60	0.10	0.38	23274
26. Wholesale trade	1.00	1.00	0.60	0.03	-0.09	27859
27. Retail food stores	1.00	1.00	0.60	0.03	0.36	17927
28. Retail gas&service	1.00	1.00	0.60	0.10	0.81	13416
29. Other retail	1.00	1.00	0.60	0.06	0.16	42141
30. Banking	1.00	1.00	0.50	0.13	-0.29	9472
31. Credit agencies	1.00	1.00	0.50	0.19	0.23	2422
32. Insurance agents&brkr	1.00	1.00	0.50	0.39	-2.00	2525
33. Real estt.agents&brkr	1.00	1.00	0.50	0.43	-0.41	3435
34. All other fins,insur	1.00	1.00	0.50	1.56	-0.99	4230
35. Hotels&other lodging	1.00	1.00	0.40	0.23	-0.12	6511
36. Medical&legal svcs.	1.00	1.00	0.40	0.31	1.75	41476
37. Educational services	1.00	1.00	0.20	0.13	0.41	4899
38. Other personal svcs.	1.00	1.00	0.40	0.16	0.30	6510
39. Railroads	1.00	1.00	0.80	9.58	-141.04	1169
40. Trucking&warehousing	1.00	0.50	0.80	0.12	-0.30	8174
41. All other transport	1.00	1.00	0.80	0.26	1.98	1414
42. Communication	1.00	1.00	0.50	1.29	2.98	9094
43. Electric utilities	1.00	1.00	2.50	0.18	-0.12	3229
44. Gas utilities	1.00	1.00	2.50	0.16	-0.24	3229
45. Water&sanitary utils	1.00	1.00	0.20	0.28	-0.11	3229

Notes: The sectors shown here are at a somewhat higher level of aggregation than those in Loviscek et. al. (1979); tax packages are represented by the decreases they provide for, as a percentage of gross sales. Source of employment data is *County Business Patterns*(1981). (Employment per sector in the FY 86 economy is assumed similar to that in the 1981 economy.)

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APPENDIX II

Algebraic Derivation of Price and Quantity Effects

This appendix details the derivation and price and quantity effects. The notation introduced in section 2 is used. In addition, the assumptions referred to are those given in the text of sections 2 and 3.

By the definition of V, we have

$$X_j^p = V_j^p + \sum_i M_{ij}^p \; \forall \, j \qquad (1)$$

By assumptions 3 and 4, all components of V_j except before-tax profits are unchanged. Thus, the behavioral postulate implies:

$$V_j^a = V_j^b - \alpha_j T_j \; \forall \, j . \qquad (2)$$

By assumptions 2 and 3,

$$s_i = X_i^a / X_i^b \quad . \qquad (3)$$

and

$$s_i = M_{ij}^a / M_{ij}^b . \qquad (4)$$

Now,

$$\begin{aligned} s_j X_j^b &= X_j^a && \text{by 3} \\ &= V_j^a + \sum_i M_{ij}^a && \text{by 1} \\ &= V_j^b - \alpha_j T_j + \sum_i s_i M_{ij}^b && \text{by 2 and 4} \end{aligned}$$

Subtracting $\sum_i s_i M_{ij}^b$, then dividing by X_j^b,

$$s_j - \sum_i s_i N M_{ij}^b = N V_j^b - \alpha_j N T_j \; \forall j$$

or, in matrix notation,

$$s - s\,NM^b = NV^b - \langle \alpha \rangle \, NT$$

Factoring out $(I - NM^b)$ on the left hand side and post-multiplying by its inverse, we obtain an expression which yields a vector with one item per sector, where each item is the ratio of new to old prices for the corresponding sector:

$$s = (NV^b - \langle \alpha \rangle \, NT)\,(I - NM^b)^{-1} \qquad (5)$$

Now, (5) corresponds to (8) in Aaron (1968) without a value-added adjustment, which we omit because we do not consider, as did Aaron, replacement of the tax revenue lost in period 2 by a value-added tax.

Now that price effects are defined, we let η denote a postulated vector of demand elasticities by sector,[1] q the vectors of ratios of new to old final demands, and we substitute the vector of price effects derived above, s, into the equation,

$$\langle q \rangle = \langle \eta \rangle \langle s \rangle$$

to obtain:

$$\langle q \rangle = \langle \eta \rangle \langle (NV^b - \langle \alpha \rangle \, NT)(I - NM^b)^{-1} \rangle$$

The impact of such changes in final de-

mands on gross outputs can now be obtained in the traditional fashion through the use of the total requirements table given by the expression:

$$X = (I - NM^b)^{-1} Y$$

where Y is the vector of final demands following the quantity reaction to changed prices.

We may by substitution of the above equations thus express the vector of total sectoral outputs in period four, X, as a function of FD, the vector of actual pre-tax-change final demands:

$$X = (I - NM^b)^{-1} \langle FD \rangle \langle\langle \eta \rangle\rangle \langle (NV^b - \langle \alpha \rangle NT) \cdot (I - NM^b)^{-1} \rangle\rangle \iota \quad (6)$$

Note that the above is in old (period one) prices.

FOOTNOTE

[1]Note that the assumed demand elasticities specify changes in demand faced by each sector, and not changes in demand for generic commodities. The distinction is of consequence in the case of commodities which are imported, as well as being produced domestically.